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Seismic response of bridge piers on elasto-plastic Winkler foundation allowed to uplift

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Abstract

When the bridge piers with shallow foundation are subject to intensive earthquake excitations, uplift of foundations will occur and the foundation soil will partly become plastic. It is very difficult to use an accurate method to simulate the uplifting and yielding of supporting soil. An improved Winkler foundation model, which could be used to consider the uplift and yield, was employed in the analysis. The 1940 El Centro earthquake record is inputted to a rigid pier with shallow foundation so that the non-linear history response is obtained. From the non-linear analysis, it is concluded that the non-linear effect is very remarkable when uplifting and yielding of supporting soil are considered. Compared with the linear analysis, the stiffness of bridge pier–soil system degrades in each cycle after considering uplifting and yielding. It is shown that the non-linear analysis can get larger rotational angles and smaller bending moments compared with the linear analysis.

1. Introduction

Commonly, when seismic response analysis of bridge piers is carried out, foundations and soils are considered to be cohesive together completely, that is to say, disconnection between foundations and soils is not taken into account. In fact, shallow foundation bridge piers are supported on foundations by gravity action mainly. When the seismic action is large enough, the overturning moment caused by earthquakes will be more than the righting moment provided by gravity. The base bottom of bridge piers with shallow foundations will disconnect to soils, even overturn occurs. On the other hand, when the foundation of bridge piers is uplifted, the local concentrating phenomenon of stress will make the soil of foundation verge enter into plasticity.

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Therefore, when seismic response analysis of bridge piers is carried out under strong earthquakes, it is necessary to consider the uplifting and plasticity of supporting soil.

After considering the uplift and yield under strong earthquakes, the traditional Winkler foundation model is no longer adaptable. In order to study the effect of uplifting, Psycharis and Jennings [1], and Yim and Chopra [2] adapted the Extensive Winkler Foundation Model allowed uplifting in their analysis. On the basis of Psycharis's Extensive Winkler Distributing Component Model and Bi-Component Model, the seismic response of buildings on the flexible foundation was studied [3]. Wolf and Skrikerud [4] studied the effect of soil yielding on structural response under the condition of uplifting by using the distributing elasto-plastic spring. Izumi et al. [5] did research on the swinging vibration of buildings after considering uplifting and yielding of foundation soil. But the above approaches used in their analysis are too simple so that it is unable to reflect the actual hysteretic characteristics of foundation. Rongchang Wang et al. [6] studied the non-linear seismic response of multi-shear-type structures by using a double-linear rotation-type spring model. However, these models mentioned above are not proved by any tests. This paper proposes a practical foundation model on the basis of tests in order to make up the lack of the previous models.

In order to get an improved Winkler foundation model, the authors carried out the tests of hysteretic characteristics under the horizontal repeated loading in the laboratory. A new elasto-plastic Winkler foundation model-allowed uplifting was presented [7]. The hysteretic characteristics of rigid shallow foundation bridge piers were worked out by using this model. The hysteretic curves, simulated by using numerical method, match the test results well, which proves the adaptability of the elasto-plastic Winkler foundation model-allowed uplifting [8].

By using this model, this paper will investigate the analysis method of seismic response of bridge piers with shallow foundations and study the effect of foundation uplifting and plasticity on the seismic response of bridge piers.

2. Elasto-plastic Winkler foundation model-allowed uplifting

The elasto-plastic Winkler foundation model-allowed uplifting is shown in Fig. 1. It is well known that for the traditional Winkler foundation model, the $M-\theta$ relation (between moment and rotation) is linear (shown in Fig. 2).

If the foundation allows uplifting, then, $M-\theta$ relation is non-linear (see Fig. 3) when a large moment is suffered [9,10].

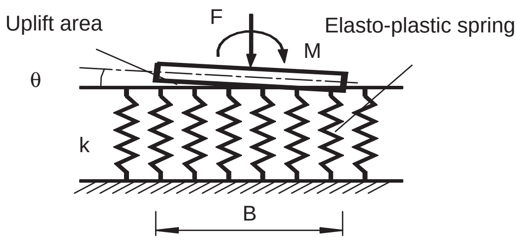

Fig. 1. Elasto-plastic Winkler model.

But if only the effect of foundation uplifting is considered, the $M-\theta$ relationship is non-linear and elastic; the unloading and loading curves are coincident and no hysteretic loop is formed. However, from the test results, it can be seen that the hysteretic loop exists in the course of repeated loading (shown in Fig. 4), which is obviously caused by foundation plasticity. Further research demonstrates that the hysteretic loop of $M-\theta$ relationship is mainly produced by the

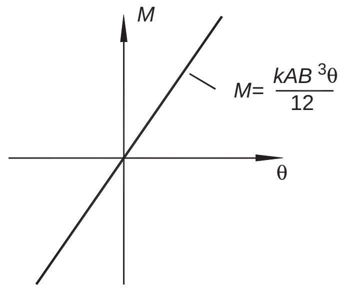

Fig. 2. $M-\theta$ relationship of the traditional Winkler model.

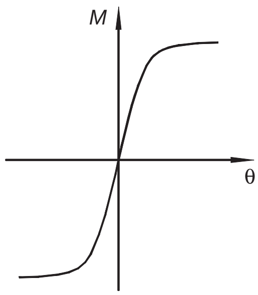

Fig. 3. $M-\theta$ relationship of the elasto-plastic Winkler model allowed to uplift.

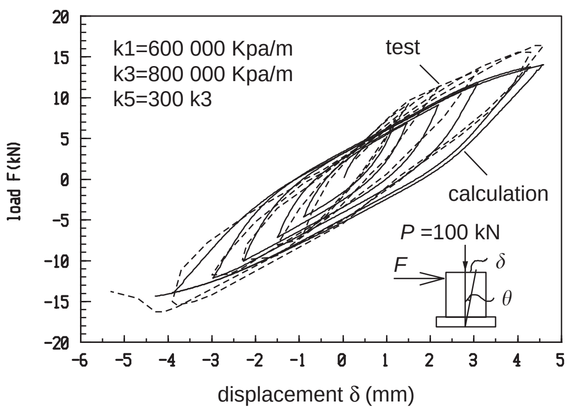

Fig. 4. Load–displacement relationship.

non-overlap of foundation compression and resilience. The compression and resilience curves obtained from the test are shown in Fig. 5 [8].

According to actual compression and resilience curves, the authors of this paper propose an elasto-plastic Winkler foundation model composed by distributing elasto-plastic springs (shown in Fig. 6). In this model, the stiffness values of loading, unloading and reloading are determined by five parameters: start-up loading modulus k_1, unloading moduli k_5 and k_2, reloading modulus k_3 as well as failure load p_y. Start-up loading is along the broken line OAB. When it comes to unloading, if, unloading begins from framework curve (sections OA or AB) or from reloading state, it carries on according to the following rules: it is unloaded by modulus k_5 to half of the present loads, then it is unloaded by unloading modulus k_2. Thus, reloading modulus k_3 can be formulated according to unloading moduli k_5 and k_2. If reloading is done at point G, then the reloading line passes through the central point H of Section DF and reloading modulus k_3 can be thus formulated. If reloading is carried out at any arbitrary point of sections DF or FG, reloading line is paralleled with GH and reloading can also be done by modulus k_3. In addition, independent of non-cohesive soil or cohesive soil, this elasto-plastic spring can only be compressed, but not pulled since the cohesive action between foundations and soils is not considered.

Because the repeated action of dead loads and live loads on bridge piers generally has been carried out before earthquakes take place, the loading modulus of foundation springs should be calculated from k_3, not from k_1.

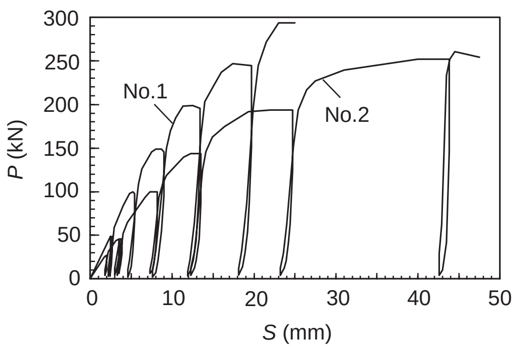

Fig. 5. Compression and resilience curves.

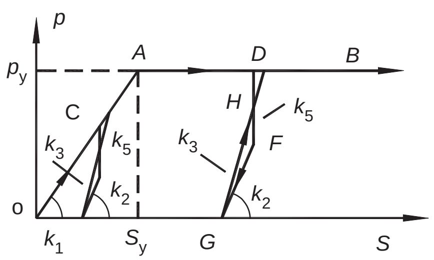

Fig. 6. Elasto-plastic Winkler model proposed by the authors.

After the dual non-linear effect of uplifting and plasticity is considered, the $M-\theta$ relation is hard to formulate with a revealed hysteretic model because of the complexity of hysteretic regularity. Thus, this paper will not apply the approach by the revealed $M-\theta$ relation, but to find the solution directly from the elasto-plastic Winkler foundation model. That is to say, in every time step of direct integration, that tangent stiffness and restoring moment of non-linear rotation-type springs are determined by the present rotation and strain history of foundation springs.

3. Motion equation and solution method

For the gravity bridge pier with lower body, the elastic deformation of piers can be neglected, so it can be simplified as a single degree-of-freedom system. The s.d.o.f. model of the pier and foundation system is shown in Fig. 7.

In this model, m_d is the lumped mass of superstructures; M_c the equivalent mass of the total mass of superstructures and the pier; H the total height of bridge piers and h_c the height from the point of equivalent mass m_d to the base bottom.

In the analysis of soil–structure interaction systems, a perfect dynamic model of bridge pier system should include mass, stiffness and damping of supporting soil. But, in this paper, the mass of supporting soil is neglected in order to simplify the analysis. As a matter of fact, in the fundamental vibration mode, the vibration of bridge piers is predominant and the supporting soil only produces corresponding static deformation. So the inertia mass of the soil can be neglected. As for the damping of supporting soil, we use the hysteretic damping to express the damping of soil, which increases with the increase of the soil strain.

When the rotational angle θ of bridge piers winding the central point O of base bottom is used as a generalized co-ordinate, then the incremental differential motion equation of the s.d.o.f. system without considering the gravity influence can be written as

$$J_0\,\Delta\ddot{\theta} + C\,\Delta\dot{\theta} + \Delta M(\theta) = -M_c h\,\Delta a_g(t), \tag{1}$$

where J_0 is the rotational inertia round the rotating center O; C is the damping coefficient, $C = 2\xi\sqrt{J_0 K_0}$; ξ is the damping ratio; K_0 the start-up tangent stiffness; $M(\theta)$ is the restoring moment and it is determined by the tangent stiffness $K(\theta)$ and the histories of angle θ; Δ is the symbol of increment and $a_g(t)$ is the horizontal seismic acceleration input from base bottom.

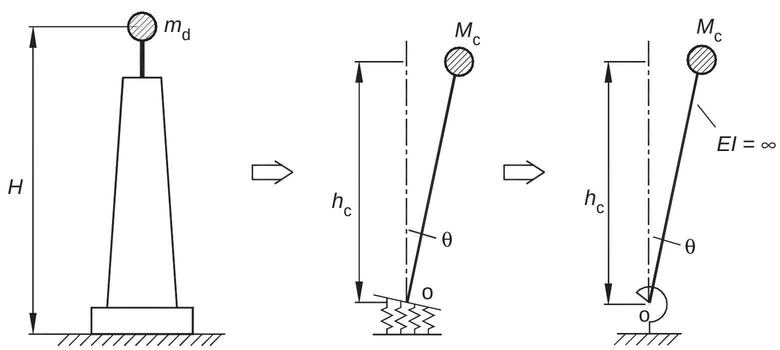

Fig. 7. s.d.o.f. Model of rigid bridge model on sand soils.

The magnitude of vertical force has a great effect on the uplifting and plasticity of supporting soil. Although not directly reflected in Eq. (1), it is indirectly reflected through tangent stiffness $K(\theta)$ of rotational springs and the restoring moment.

The solution to Eq. (1) is carried out by the Wilson θ method. In every time step of integration, the tangent stiffness $K(\theta)$ is determined by the present rotation and loading history. So the present base-bottom moment $M(\theta)$ can also be determined by the present rotation and loading history. In order to decide the tangent stiffness $K(\theta)$ used in the next step, it is necessary to distinguish between loading and unloading because the tangent stiffness of loading or unloading is not the same. For the convenience of formulating explanation, this paper defines the clockwise rotation of bridge piers as loading, and the reverse is unloading. Based on this assumption, the angular velocity $\dot{\theta}$ can be used to decide loading or unloading so that the increment $\Delta\theta$ is determined; and the determination of positive or negative symbols of $\Delta\theta$ and the calculation method of tangent stiffness are as follows.

If $\dot{\theta} > 0$, it is loading and $\Delta\theta$ is positive.

If $\dot{\theta} < 0$, it is unloading and $\Delta\theta$ is negative.

Thus, by calculating the moment $M(\theta + \Delta\theta)$ at the point $\theta + \Delta\theta$, the secant stiffness can be expressed as

$$K(\theta) = \frac{M(\theta + \Delta\theta) - M(\theta)}{\Delta\theta}. \tag{2}$$

When $\Delta\theta$ is infinitely small, the secant stiffness is equal to the tangent stiffness. In actual analysis, $\Delta\theta$ is considered to be relatively small. Loading or unloading must be determined in every time step of direct integration and a new stiffness $K(\theta)$ must be calculated at the same time.

The 1940 El Centro NS acceleration record is adapted in the analysis and the value of the record is adjusted according to the seismic intensity. The maximum accelerations are $0.1g$ at 7°, $0.2g$ at 8° and $0.4g$ at 9° of earthquake intensity.

4. Example

The height of the bridge pier in this example is $H = 10\,\text{m}$, lump mass of pier top $m_d = 340\,\text{t}$. Dimension of base bottom: the longitudinal B is 4.66 m, the transverse A is 5.24 m.

Under the 1940 El Centro earthquake input, the comparison of the linear and non-linear time response between the rotation and the moment of the base bottom is shown in Fig. 8. The comparison of non-linear time responses in different earthquake intensities is shown in Fig. 9. The $M - \theta$ hysteretic curves for various intensities are shown in Fig. 10. The maximum values of foundation rotational angle and moment are listed in Table 1.

From the comparison of linear and non-linear analysis results shown in Fig. 8 and in Table 1, it can be found that the foundation plasticity and uplift make the non-linear response distinctly different from the linear response. The rotational angle obtained from non-linear analysis is obviously bigger than that of linear analysis while the bending moment of base bottom obtained from non-linear analysis is clearly smaller than that of linear analysis.

The non-linear analysis results (shown in Fig. 9) show that with the increase of earthquake intensity, the incremental amplitude of rotation enlarges very fast. The amplitude of rotation at 9°

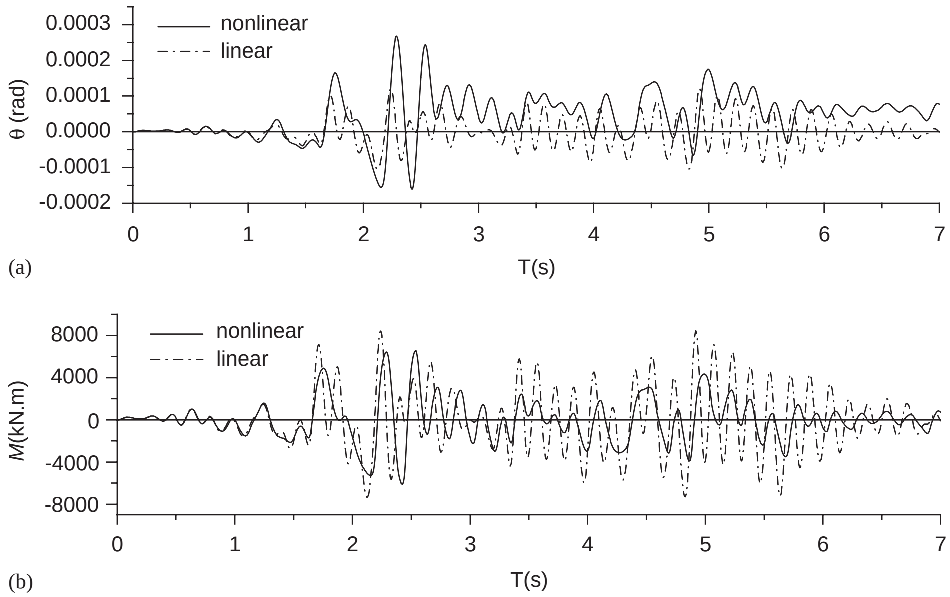

Fig. 8. Comparison of linear and non-linear responses (7°): (a) comparison of linear and non-linear rotational angle; (b) comparison of linear and non-linear bending moment.

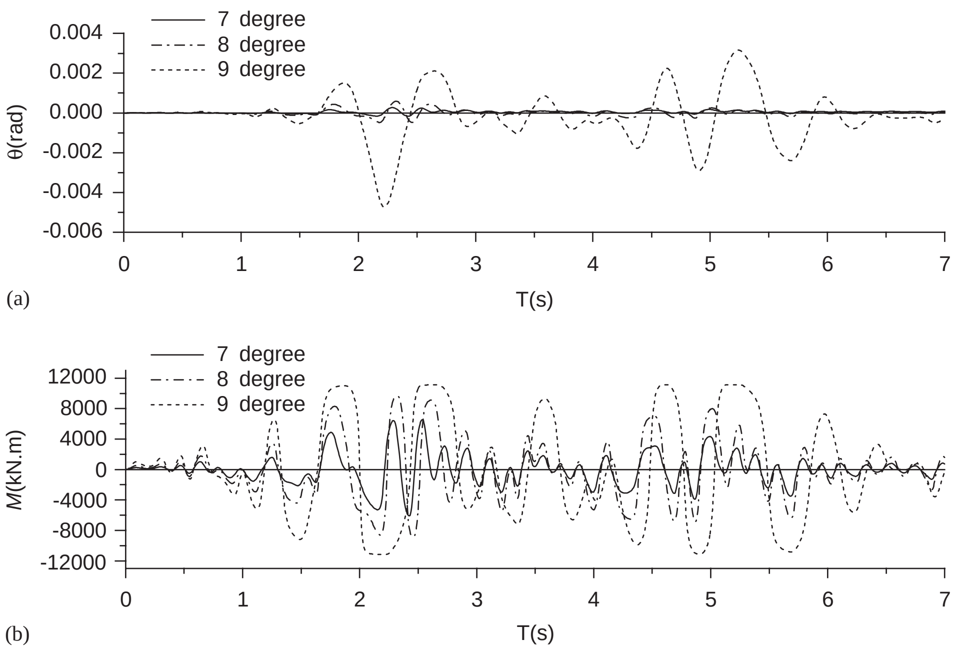

Fig. 9. Comparison of non-linear responses in different earthquake intensities: (a) comparison of rotational angles; (b) comparison of bending moments.

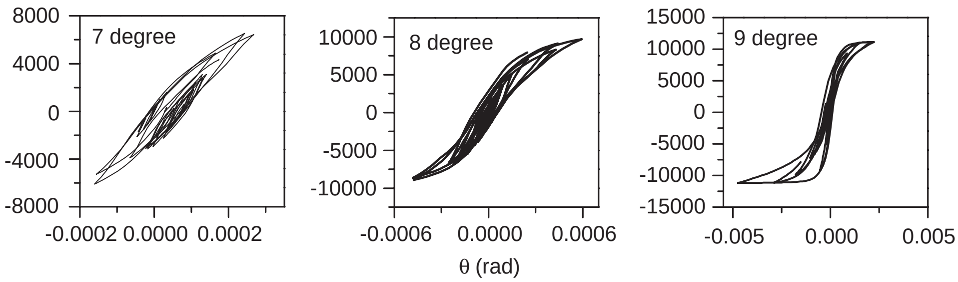

Fig. 10. $M-\theta$ hysteretic curves.

Table 1
Maximum values of rotational angle and moment

Earthquake intensity	Rotational angle θ (rad $\times 10^{-4}$)		Ratio of non-linear to linear	Bending moment M (kN m)		Ratio of non-linear to linear
	Linear	Non-linear		Linear	Non-linear	
7°	1.21	2.68	2.2	8454	6540	0.8
8°	2.42	5.93	2.5	16908	9710	0.6
9°	4.84	47.4	9.8	33816	11168	0.3

is about 8 times the amplitude at 8°. As for the moment of the base bottom, the increment is relatively small with the increase of intensity.

Because the non-linearity of soils makes the foundation springs soften, the vibration period of the non-linear response is bigger than the period of the linear response. Further, the vibration period will increase with the increase of intensity.

From the $M-\theta$ hysteretic curves shown in Fig. 10, it can be seen that at the intensity of 7°, the $M-\theta$ hysteretic curve is spindle-shaped, which mainly reflects the effect of foundation plasticity. At the intensity of 8°, the $M-\theta$ hysteretic curve is S-shaped, which embodies the uplifting effect of foundation besides the plastic energy consumption of supporting soil. At the intensity of 9°, the S-shape of hysteretic curve is more obvious, which means that the foundation uplifting plays a main role.

It can be obviously seen that foundation uplifting and plasticity have a significant effect on the reduction of seismic force of bridge piers. The foundation-allowed uplifting and yielding in design will protect bridge piers and superstructures. The value of reduction of seismic force increases with the increase of seismic intensity. From the above results, we can qualitatively explain why some of the bridge piers had only a slight damage at the ultimate earthquake areas of the Tangshan Earthquake of China in 1976.

It should be noted that foundation plasticity is advantageous in reducing the seismic force of bridge piers and superstructures, but the plasticity of foundation will cause bridge piers to deviate and produce permanent tilt distortion away from the original position (see Figs. 8 and 9). Therefore, the permanent deformation of bridge structures should be controlled within permission in order that some in-service performance can be maintained after intensive earthquakes. In addition, the maximum deformation of bridge piers should be smaller than ultimate deformation in order to avoid girder falling and other damage of superstructures.

5. Conclusions

This paper presented a practical analysis model of a single degree-of-freedom (s.d.o.f.) system of bridge piers for seismic response analysis. The interaction between bridge piers and supporting soil was simulated by the improved elasto-plastic Winkler model in which the uplifting effect of foundations can be considered. Using this analysis model, the non-linear seismic response of a practical bridge pier was studied under the 1940 El Centro earthquake record input. We can make the following conclusions from the analysis.

The non-linear seismic response of bridge piers is distinctly different from that of the linear response. There is a great difference whether it is in vibration amplitude or in frequency property. The non-linear properties of foundations make the stiffness of the structure low, the response of rotational angle increase and the response of bending moment decrease.

Permitting uplifting and plasticity of foundations in design can reduce the seismic load acting on the pier body and play an advantageous role in the seismic ability of bridge piers. This is the reason why some of the bridge piers had only a slight damage at the ultimate earthquake areas in the 1976 Tangshan Earthquake of China.

On the basis of this paper, further research can be made for the seismic response of a soft bridge pier on the elasto-plastic Winkler foundation-allowed uplifting.

Acknowledgements

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